Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
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7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(4 marks)**

Differentiate the following with respect to x: (Do not simplify your answers)

(a) $y = 5\sin^2(3x)$ **(2 marks)**

(b) $y = 3x(8x^4 - 2x)^5$ **(2 marks)**

Question 2   **(3 marks)**

Given the function $y = -\dfrac{1}{\sqrt{4x+3}}$ then $\dfrac{dy}{dx} = \dfrac{m}{(4x+3)^n}$

(a) State the values of m and n.   (2 marks)

(b) Determine the instantaneous rate of change of y when $x = 1.5$.   (1 mark)

See next page

Question 3 **(4 marks)**

A spherical balloon is leaking gas. Use the Incremental formula to estimate the percentage change in the radius if its volume decreases from $800\ cm^3$ to $788\ cm^3$.
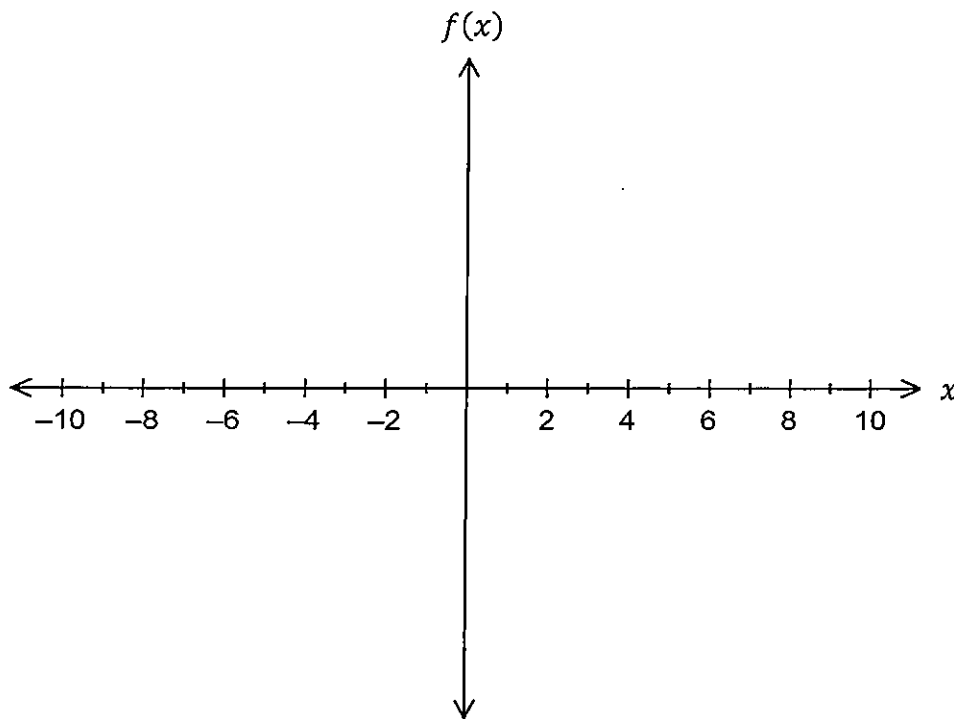
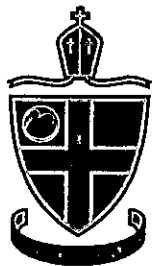**Question 4**   **(4 marks)**

Sketch a function $y = f(x)$ with all of the following features. Label the critical features clearly.

- $f(-2) = f(6) = f(8) = 0$
- $f'(1) = f'(7) = 0$
- $f''(4) = 0$
- $f''(x) < 0 \ for \ x < 4$ only

Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

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Formula Sheet (retained from Section One)

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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 5   **(6 marks)**

A particle is moving along a straight line that runs in an east-west direction. Its position function $s(t)$ at time t is given by $s(t) = \dfrac{t^2+1}{t^4+1}$

(a)   Determine the velocity function of the particle.   (1 mark)

(b)   The particle is moving in an easterly direction when the velocity is positive. Use the graph of the velocity function on your calculator to determine when the particle is moving in a westerly direction.   (2 marks)

(c)   Use the graph of the velocity function to determine the maximum speed of the particle and when it is attained.   (2 marks)

(d)   Calculate the position of the particle at the time when the maximum speed is attained.   (1 mark)

See next page

Question 6   **(3 marks)**

If $f(x) = (1 - x^2)^{\frac{3}{2}}$, then determine,

(a) $f''(x)$.

                                                 (1 mark)

(b)   the domain of $f''(x)$.

                                                 (2 marks)

Question 7   **(5 marks)**

The function h is defined so that $h(0) = 2$ and $h'(0) = 1$.

(a)   If $f(x) = x \times h(x)$, then determine $f'(0)$.   **(2 marks)**

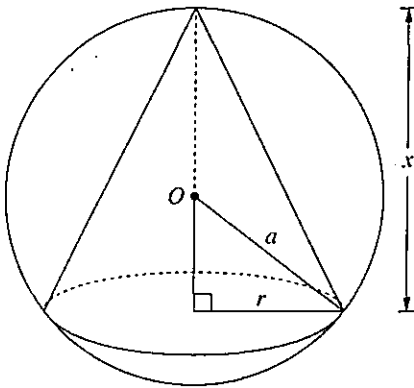(b)   If $g(x) = h(x) - \frac{1}{h(x)}$, then determine $g'(0)$.   **(3 marks)**

Question 8   **(6 marks)**

A cone is inscribed in a sphere of radius a, centred at O. The height of the cone is x and the radius of the base is r, as shown in the diagram.

(a)   Show that the volume, V, of the cone is given by $V = \frac{1}{3}\pi(2ax^2 - x^3)$ (2 marks)
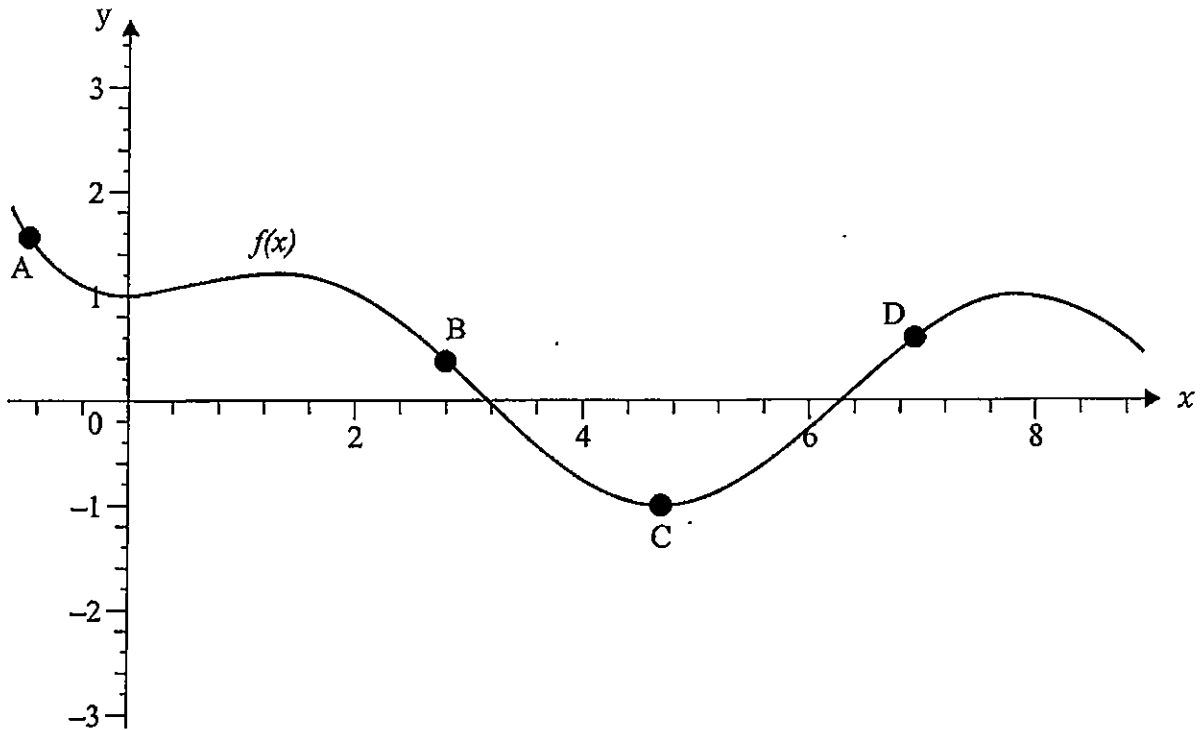

(b)   Find the value of x for which the volume of the cone is a maximum. Verify that your value of x gives the maximum value.   (4 marks)

Question 9   **(10 marks)**

The graph of a function $f(x)$ is given on the interval $-1 \le x \le 9$.

(a)   A, B, C, D are four points on the graph of $f(x)$. Determine whether the first and second derivatives are positive, negative or equal to zero at these points. Record your findings in the table below.   (4 marks)

Point	$f'(x)$	$f''(x)$
A		
B		
C		
D		

(b)   Indicate on the graph of f above the other two inflection points and label them E and F.   (2 marks)

(c)   Sketch the graph of $f'(x)$ on the same axes of the graph of $f(x)$ above.
                                                                          (4 marks)

End of questions

Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name — Solutions —

Teacher's name

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Question 1 (4 marks)

Differentiate the following with respect to x: (Do not simplify your answers)

(a) $y = 5\sin^2(3x)$ (2 marks)

$$\frac{dy}{dx} = 5(2) \sin(3x) \cos(3x) \times 3$$

$$\frac{dy}{dx} = 30 \sin(3x) \cos(3x)$$

✓✓

$\begin{bmatrix} \text{take } 1mk \text{ off} \\ \text{for each error} \\ \text{until zero} \end{bmatrix}$

(b) $y = 3x(8x^4 - 2x)^5$ (2 marks)

$$\frac{dy}{dx} = 3(8x^4-2x)^5 + 3x(5)(8x^4-2x)^4(32x^3-2)$$

✓✓

$\begin{bmatrix} \text{take } 1mk \text{ off} \\ \text{for each error} \\ \text{until zero} \end{bmatrix}$

4

Question 2   **(3 marks)**

Given the function $y = -\dfrac{1}{\sqrt{4x+3}}$ then $\dfrac{dy}{dx} = \dfrac{m}{(4x+3)^n}$

(a)   State the values of m and n.   (2 marks)

$$y = -(4x+3)^{-\frac{1}{2}} \qquad \therefore \frac{dy}{dx} = \frac{1}{2}(4x+3)^{-\frac{3}{2}} \times (4)$$

$$\frac{dy}{dx} = \frac{2}{(4x+3)^{\frac{3}{2}}}$$

$$\therefore \quad m = 2 \qquad \checkmark \quad (m \text{ value})$$

$$n = \frac{3}{2} \qquad \checkmark \quad (n \text{ value})$$

(b)   Determine the instantaneous rate of change of y when $x = 1.5$.   (1 mark)

$$\left. \frac{dy}{dx} \right|_{x=1.5} = \frac{2}{\left(4 \times \frac{3}{2} + 3\right)^{\frac{3}{2}}}$$

$$= \frac{2}{9^{\frac{3}{2}}}$$

$$= \frac{2}{27} \qquad \checkmark \quad (\text{value})$$

3

Question 3 **(4 marks)**

A spherical balloon is leaking gas. Use the Incremental formula to estimate the percentage change in the radius if its volume decreases from $800cm^3$ to $788cm^3$.

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$\frac{\delta v}{\delta r} \approx \frac{dv}{dr}$$

$\left(\frac{dv}{dr}\right)$ ✓

$$\therefore \frac{dv}{dr} = 4\pi r^2$$

$\boxed{Also \quad \delta v = -12 \, cm^3}$

$$\delta v \approx \frac{dv}{dr} \times \delta r$$

$\left(\begin{matrix} uses \\ \% \ change \end{matrix}\right)$ ✓

$$\frac{\delta v}{V} \approx \frac{\frac{dv}{dr} \times \delta r}{V}$$

$$\delta r \approx \frac{\delta v}{1} \times \frac{dr}{dv}$$

$$\frac{\delta v}{V} \approx \frac{4\pi r^2 \times \delta r}{\frac{4}{3}\pi r^3}$$

$\Rightarrow$ Isolate $\frac{\delta r}{r}$

$\left(\begin{matrix} Sub \ for \\ \delta v, v \end{matrix}\right)$ ✓

$$\frac{-12}{800} \approx \frac{\delta r}{\frac{1}{3}r}$$

$$\frac{\delta v}{r} \approx 4\pi r^2 \frac{\delta r}{r}$$

$$\frac{-12}{800} \approx \frac{3\delta r}{r}$$

$$\therefore \frac{\delta r}{r} \approx \frac{\delta v}{4\pi r^2 \times r}$$

$$\therefore \frac{\delta r}{r} \approx \frac{-12}{2400}$$

$$\frac{\delta r}{r} \approx -\frac{1}{200}$$

$\left(correct \ \%\right)$ ✓

$$\frac{\delta r}{r} \approx -0.005$$

$$\approx 0.5\% \ reduction$$

4

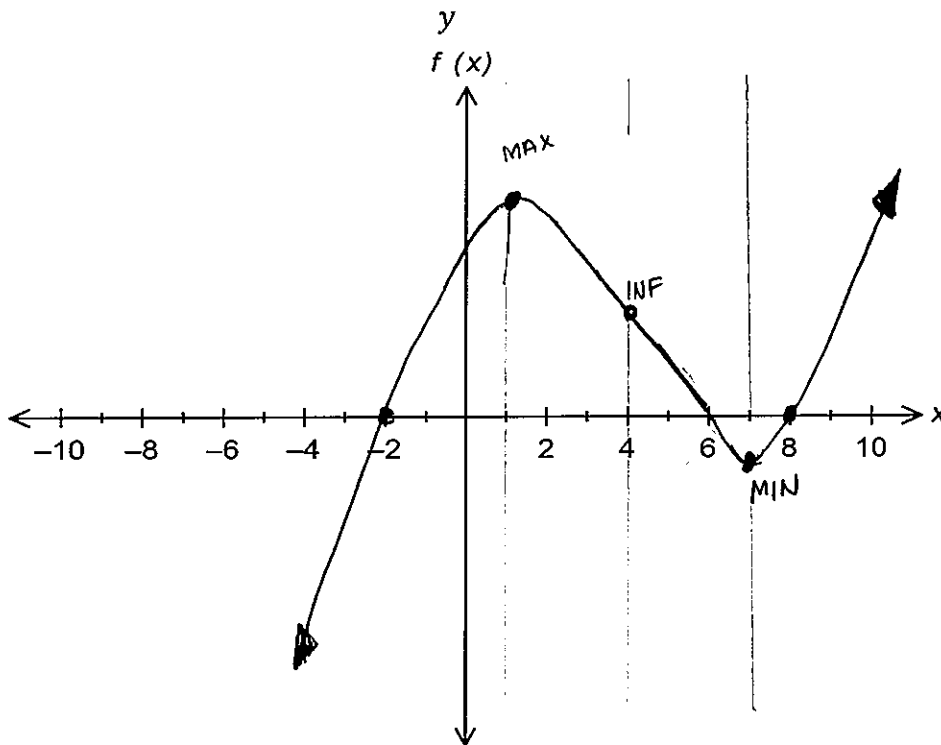**See next page**

Question 4   **(4 marks)**

Sketch a function $y = f(x)$ with all of the following features. Label the critical features clearly.

- $f(-2) = f(6) = f(8) = 0$
- $f'(1) = f'(7) = 0$
- $f''(4) = 0$
- $f''(x) < 0 \; for \; x < 4$ only

$\checkmark \; \left(Roots\right)$

$\checkmark \; \left(Inflection\right)$

$\checkmark \; \left(Max\right)$

$\checkmark \; \left(Min\right)$

End of questions

Christ Church
Grammar School

2020

TEST 1

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____ — Solutions •— _____

Teacher's name _____

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Question 5   **(6 marks)**

A particle is moving along a straight line that runs in an east-west direction. Its position function $s(t)$ at time t is given by $s(t) = \dfrac{t^2+1}{t^4+1}$

(a)   Determine the velocity function of the particle.   (1 mark)

$$v(t) = \frac{-2t^5 - 4t^3 + 2t}{\left(t^4+1\right)^2} \quad \checkmark$$

(b)   The particle is moving in an easterly direction when the velocity is positive. Use the graph of the velocity function on your calculator to determine when the particle is moving in a westerly direction.   (2 marks)
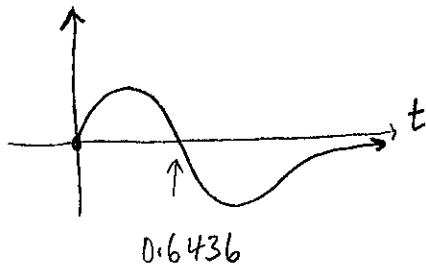
$$t > 0.6436 \quad \checkmark\!\!\checkmark$$

0.6436

(c)   Use the graph of the velocity function to determine the maximum speed of the particle and when it is attained.   (2 marks)
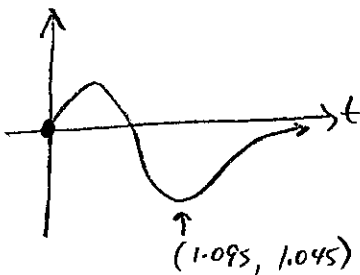
$t = 1.095 \quad \checkmark$

$\text{Max Speed} = 1.045 \quad \checkmark$

$(1.095, 1.045)$

(d)   Calculate the position of the particle at the time when the maximum speed is attained.   (1 mark)

$$s(1.095) = 0.902 \quad \checkmark$$

6

Question 6 (3 marks)

If $f(x) = (1 - x^2)^{\frac{3}{2}}$;

(a) determine $f''(x)$.

(1 mark)

$$Class\,Pad \qquad f''(x) = \frac{6x^2 - 3}{\sqrt{-x^2 + 1}} \qquad \checkmark$$

$$\underline{or} \qquad \frac{3(2x^2 - 1)}{\sqrt{-x^2 + 1}}$$

(b) determine the domain of $f''(x)$.

(2 marks)

$$\sqrt{-x^2 + 1} \; > \; 0$$

$$\therefore \; 1 - x^2 \; > \; 0$$

$$-1 \; < \; x \; < \; 1$$
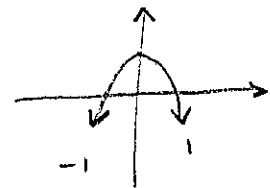
$$\checkmark \qquad\qquad \checkmark$$

lower   upper

3

Question 7   **(5 marks)**

The function h is defined so that $h(0) = 2$ and $h'(0) = 1$.

(a)   If $f(x) = x \times h(x)$, determine $f'(0)$.   (2 marks)

$$f'(x) = 1 \times h(x) + x \cdot h'(x)$$

$$f'(0) = 1 \times h(0) + 0 \times h'(0) \quad \checkmark \quad \left(\begin{array}{c}\text{Recognises}\\ \text{Product Rule}\end{array}\right)$$

$$f'(0) = 2 + 0 \times 1$$

$$f'(0) = 2 \quad \checkmark \quad \left(\begin{array}{c}\text{Sub in and}\\ \text{evaluates}\end{array}\right)$$

(b)   If $g(x) = h(x) - \frac{1}{h(x)}$ determine $g'(0)$ (3 marks)

$$g'(x) = h'(x) - \left[\frac{-1 \times h'(x)}{[h(x)]^2}\right] \quad \checkmark \quad \left(\begin{array}{c}\text{Quotient}\\ \text{Rule or other}\\ \text{for } \frac{1}{h(x)} \text{ deriv.}\end{array}\right)$$

$$g'(0) = h'(0) - \left[\frac{-1 \times h'(0)}{[h(0)]^2}\right]$$

$$g'(0) = 1 - \left[\frac{-1}{2^2}\right] \quad \checkmark \quad (\text{sub in})$$

$$g'(0) = 1 + \frac{1}{4}$$

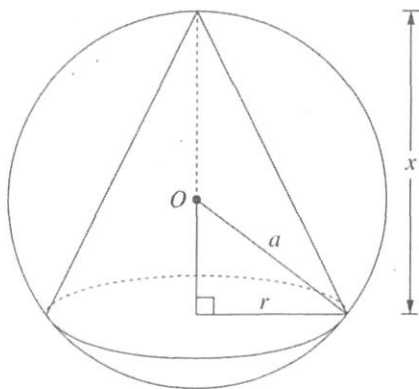$$g'(0) = \frac{5}{4} \quad \checkmark \quad (\text{correct answer})$$

Question 8   **(6 marks)**

A cone is inscribed in a sphere of radius a, centred at O. The height of the cone is x and the radius of the base is r, as shown in the diagram.

(a)   Show that the volume, V, of the cone is given by $V = \frac{1}{3}\pi(2ax^2 - x^3)$ (2 marks)

$$V = \frac{1}{3}\pi r^2 h \qquad \Rightarrow \quad r = r$$
$$h = x$$

$$V = \frac{1}{3}\pi r^2 (x)$$

$$V = \frac{1}{3}\pi (2ax - x^2)x \qquad \checkmark \text{(sub in } r^2\text{)}$$

$$V = \frac{1}{3}\pi (2ax^2 - x^3)$$

As Required

$$a^2 = r^2 + (x-a)^2$$
$$r^2 = a^2 - (x-a)^2$$
$$r^2 = a^2 - x^2 + 2ax - a^2$$
$$r^2 = 2ax - x^2$$

$\checkmark$ (finds r^2)

(b)   Find the value of x for which the volume of the cone is a maximum. Verify that your value of x gives the maximum value.   (4 marks)

$$\text{Max} \quad \frac{dv}{dx} = 0 \qquad \Rightarrow \quad \frac{-(3x^2 - 4ax)\pi}{3} = 0 \qquad \checkmark \text{(Deriv} = 0)$$

$$\text{Solve} \quad 3x^2 - 4ax = 0$$
$$x = 0, \quad \boxed{x = \frac{4a}{3}} \qquad \checkmark \left(x = \frac{4a}{3}\right)$$

reject $x = 0$ as $x = $ height.

$$\left.\frac{d^2v}{dx^2}\right|_{x = \frac{4a}{3}} = \frac{-\pi\left(6\left(\frac{4a}{3}\right) - 4a\right)}{3} \qquad \checkmark \text{(2}^{nd}\text{ Deriv check)}$$

$$= \frac{-4a\pi}{3} < 0 \qquad \therefore \text{ Max} \quad \checkmark \begin{pmatrix} \text{less} \\ \text{than 0} \\ \therefore \text{MAX} \end{pmatrix}$$
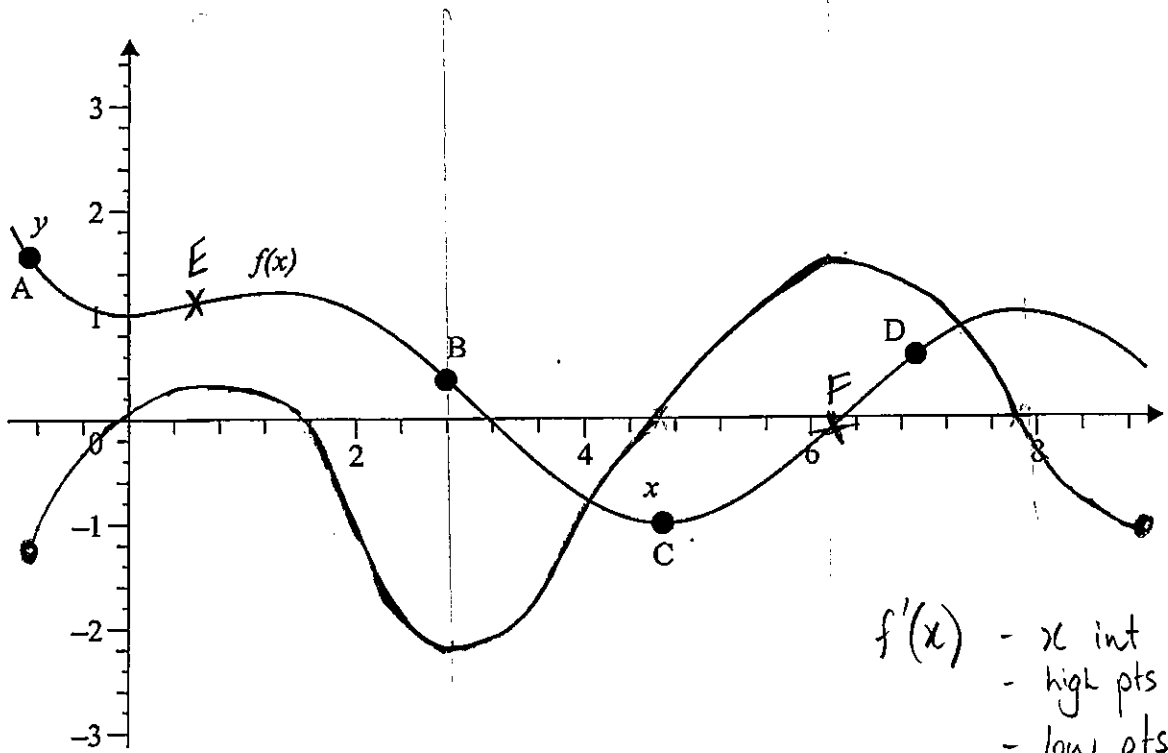
when $x = \frac{4a}{3}$

See next page

6

Question 9   **(10 marks)**

The graph of a function $f(x)$ is given on the interval $-1 \le x \le 9$.

$f'(x)$ - x int ✓
- high pts ✓
- low pts ✓
- general shape ✓

(a)   A, B, C, D are four points on the graph of $f(x)$. Determine whether the first and second derivatives are positive, negative or equal to zero at these points. Record your findings in the table below.   (4 marks)

Point	$f'(x)$	$f''(x)$	
A	NEG	POS	✓
B	NEG	ZERO	✓
C	ZERO	POS.	✓
D	POS	NEG	✓

(b)   Indicate on the graph of f above the other two inflection points and label them E and F.   (2 marks)

✓ ✓   (on Graph)

(c)   Sketch the graph of $f'(x)$ on the same axes of the graph of $f(x)$ above.

(See Graph) (4 marks)

End of questions